

BAB V

PENUTUP

A. Kesimpulan

Berdasarkan uraian pembahasan sebelumnya, maka dapat ditarik kesimpulan sebagai berikut:

- a. Diperoleh model *predator-prey* antara populasi burung hantu, padi, dan tikus dengan kontrol berupa perangkap tikus

$$\begin{aligned}\frac{dP}{dt} &= \alpha P \left(1 - \frac{P}{K}\right) - \beta PT \\ \frac{dT}{dt} &= \delta PT - \gamma BT - \mu T - uT \\ \frac{dB}{dt} &= \sigma BT \left(\frac{B}{A} - 1\right) - \theta B\end{aligned}$$

- b. Terdapat lima titik ekuilibrium yang diperoleh dari model *predator-prey* antara populasi padi, tikus, dan burung hantu tanpa kontrol ($u = 0$)

- Titik Ekuilibrium Pertama (tidak stabil)

$$E_1 = (0,0,0)$$

- Titik Ekuilibrium Kedua (stabil bersyarat)

$$E_2 = (K, 0, 0)$$

- Titik Ekuilibrium Ketiga (stabil bersyarat)

$$E_3 = \left(\frac{\mu}{\delta}, \frac{\alpha(K\delta - \mu)}{K\beta\delta}, 0\right)$$

- Titik Ekuilibrium Keempat (tidak stabil)

$$E_4 = \left(\frac{\left(\frac{1-\alpha\mu\sigma + c_1 + \sqrt{c_2 - c_3}}{2}\right)\gamma + \mu}{\delta}, -\frac{\mu A}{\sigma\left(-\frac{1-\alpha\mu\sigma + c_1 + \sqrt{c_2 - c_3}}{2}\right) + A}, \frac{1-\alpha\mu\sigma + c_1 + \sqrt{c_2 - c_3}}{2\gamma\alpha\sigma}\right)$$

- Titik Ekuilibrium Kelima (tidak stabil)

$$E_5 = \left(\frac{\left(\frac{1-\alpha\mu\sigma + c_1 - \sqrt{c_2 - c_3}}{2}\right)\gamma + \mu}{\delta}, -\frac{\mu A}{\sigma\left(-\frac{1-\alpha\mu\sigma + c_1 - \sqrt{c_2 - c_3}}{2}\right) + A}, \frac{1-\alpha\mu\sigma + c_1 - \sqrt{c_2 - c_3}}{2\gamma\alpha\sigma}\right)$$

- c. Diperoleh $u^* = \min \left\{ \max \left\{ 0, \frac{\lambda_2 T}{c_2} \right\}, 1 \right\}$ yang meminimumkan jumlah populasi tikus yang memangsa padi dan biaya pemasangan perangkap yang dikeluarkan untuk mengendalikan tikus serta yang memenuhi model *predator-prey* antara populasi padi, tikus, dan burung hantu. Simulasi numerik menggambarkan bahwa penerapan kontrol perangkap tikus dapat meningkatkan populasi padi.

B. Saran

Penelitian ini fokus pada pengkajian model *predator-prey* antara populasi padi, tikus, dan burung hantu dengan diberikan kontrol yang berupa perangkap tikus. Penelitian selanjutnya disarankan agar mempertimbangkan penambahan respon Holling tipe II agar menghasilkan model yang rasional.



DAFTAR PUSTAKA

- Anton, H., & Rorres, C. (2004). *Aljabar Linear Elementer* (Edisi ke-8). Penerbit Erlangga.
- Arianto, A., & Qodir, F. (2005). Alat Perangkap Tikus Elektronik. *Ilmiah Semesta Teknik*, 8(2), 192–201.
- Boyce, W. E., & DiPrima, R. C. (2009). *Elementary Differential Equations and Boundary Value Problems* (9 ed.).
- Cahyaningtias, S. (2016). Aplikasi Kriteria Routh-Hurwitz pada Kestabilan Model Interaksi Padi-Hama. *Wahana*, 66(1), 69–73. <https://doi.org/10.36456/wahana.v66i1.491>
- Dopita, E. (2022). 7 Fakta Menarik Burung Hantu, dapat Menelan Mangsa Secara Utuh. Kompas.com. <https://www.kompas.com/homey/read/2022/10/31/140000376/7-fakta-menarik-burung-hantu-dapat-menelan-mangsa-secara-utuh>. Diakses pada tanggal 10 Oktober 2024.
- Gao, S., Hu, J., & Yu, Y. (2018). Iterative Methods for Polynomial Equations Based on Vieta ' s Theorem. *Advances in Computer Science Research*, 83, 107–111.
- Junaedi, J. (2024). Petani di Garut Pasang Rumah Burung Hantu, Solusi Nyata Atasi Hama Tikus. Kompasiana.com. <https://www.kompasiana.com/jujunjunaedia37689/66d6481834777c5d423c7164/petani-di-garut-pasang-rumah-burung-hantu-solusi-nyata-atasi-hama-tikus>. Diakses pada tanggal 28 September 2024.
- Kot, M. (2001). *Element of Mathematical Ecology*. University Press.
- Lenhart, S., & Workman, J. (2007a). Optimal Control Applied to Biological Models. In *CRC Press, Taylor & Francis Group*. CRC Press, Taylor & Francis Group. <https://doi.org/10.1201/9781420011418>
- Listyana, L., Hartono, & Krisnawan, K. P. (2016). Analisis Bifurkasi pada Model Matematis Predator-Prey dengan Dua Predator. *Jurnal Kajian dan Terapan Matematika*, 1–12.
- Munir, R. (2015). *Metode Numerik* (Edisi Ke-4). Informatika Bandung.
- Ndii, M. Z. (2022). *Pemodelan Matematika*. PT. Nasya Expanding Management.
- Nurmadhani, N., & Faisol. (2022). Penerapan Model Pertumbuhan Logistik Dalam Memproyeksikan Jumlah Penduduk Di Kabupaten Sumenep. *Jurnal Edukasi dan Sains Matematika (JES-MAT)*, 8(2), 181–192. <https://doi.org/10.25134/jes-mat.v8i2.5436>
- Olsder, G. J., Van Der Wode, J. W., Maks, J. G., & Jeltsema, D. (2011). *Mathematical Systems Theory* (Edisi Ke-4). VVSd.
- Padilah, T. N., Sari, B. N., & Hannie, H. (2018). Model matematis predator-prey

- tanaman padi, hama penggerek batang, tikus, dan wereng batang coklat di Karawang. *Pythagoras: Jurnal Pendidikan Matematika*, 13(1), 52–62. <https://doi.org/10.21831/pg.v13i1.16880>
- Panigoro, H. S., & Savitri, D. (2020). Bifurkasi Hopf pada Model Lotka-Volterra Orde-Fraksional dengan Efek Allee Aditif pada Predator. *Jambura Journal of Biomathematics (JJBM)*, 1(1), 16–24. <https://doi.org/10.34312/jjbm.v1i1.6908>
- Porusia, M., & Darnoto, S. (2019). *Pengendalian Vektor Penyakit*. Muhammadiyah University Press.
- Pusparini, M. D., & Suratha, I. K. (2018). Efektivitas Pengendalian Hama Tikus pada Tanaman Pertanian dengan Pemanfaatan Burung Hantu Di Desa Wringinrejo Kecamatan Gambiran Kabupaten Banyuwangi, Provinsi Jawa Timur. *Jurnal Pendidikan Geografi Undiksha*, 6(2), 54–63. <https://doi.org/10.23887/jjpg.v6i2.20683>
- Ra'yan, I., Toaha, S., & Kusuma, J. (2022). Dynamics Analysis of Predator-Prey Model with Double Allee Effects and Holling Type II Functional Response. *Jurnal Matematika, Statistika dan Komputasi*, 18(3), 434–446. <https://doi.org/10.20956/j.v18i3.19237>
- Rahmawati, R., & Savitri, D. (2023). Model Lotka-Volterra yang Mempertimbangkan Efek Ketakutan pada Prey dengan Fungsi Holling Tipe II. *Ilmiah Matematika*, 11(2), 304–309.
- Renny, & Roerita, R. (2021). Model Dinamik Kontrol Optimal Predator-Prey dengan Respon Fungsional Beddington-De Angelis pada Tanaman Padi. *Matematika dan Pendidikan Matematika*, 4(1), 38–53.
- Rifandi, M., & Abdi, M. (2023). *Suatu Pengantar Persamaan Differensial Biasa*. Uwais Inspirasi Indonesia.
- Rizka, N. (2022). Model Rantai Makanan antara Tanaman Jagung, Hama Tikus, dan Burung Hantu dengan Pemasangan Perangkap Tikus. *Mathematic dan Applications*, 173–180.
- Ros, S. L. (1989). *Introduction To Ordinary Differential Equations* (Edisi ke-4). John Wiley & Sons.
- Sa'adah, N., Bakhtiar, T., & Hanum, F. (2012). *Penerapan Prinsip Maksimum Pontryagin pada Sistem Inventori-Produksi*. November, 224–235.
- Salmah. (2020). *Teori Sistem Kendali linear dan Aplikasinya*. Gadjah Mada University Press.
- Sinaga, L., Kartika, D., & Hamidah, N. (2021). *Pengantar Sistem Dinamik*. Amal Insani.
- Siregar, H. M., Priyambodo, S., & Hindayana, D. (2020). Preferensi Serangan Tikus Sawah (*Rattus argentiventer*) Terhadap Tanaman Padi. *Agroekoteknologi*,

13(1), 16–21. <https://doi.org/10.21107/agrovigor.v13i1.6249>

Siregar, I. M., & Sulardi, I. (2018). *Agribisnis Tanaman Padi* (M. Wasito (ed.)). Fakultas Ekonomi Universitas Panca budi.

Supriadin, J. (2024). *Puluhan Hektare Sawah di Garut Terserang Hama Tikus, Kerugian Capai Rp1,8 miliar*. Liputan 6. <https://www.liputan6.com/regional/read/5664755/puluhan-hektare-sawah-di-garut-terserang-hama-tikus-kerugian-capai-rp18-miliar>. Diakses pada tanggal 17 September 2024.

Wiggins, S. (2006). *Intoduction to Applied Nonlinear Dynamical System and Chaos*. Springer Science.

LAMPIRAN

Lampiran 1. Simulasi Numerik Titik Ekuilibrium Kedua

```

> restart :
>
with(linalg) : with(LinearAlgebra) : with(plots) : with(Student[VectorCalculus]) : with(DEtools) : with(plots,
implicitplot) : with(Student[CalculusI]) :

> dP := α·P·(1 - P/K) - β·P·T; dT := δ·P·T - γ·B·T - μ·T; dB := σ·B·T·(B/A - 1) - θ·B;
      dP := α P (1 - P/K) - β P T
      dT := -B T γ + P T δ - T μ
      dB := σ B T (B/A - 1) - θ B

> TK := solve({dP, dT, dB}, {P, T, B});
TK := {B=0, P=0, T=0}, {B=0, P=K, T=0}, {B = -μ/γ, P=0, T = -θ A γ / (σ (A γ + μ))}, {B=0, P = μ/δ, T =
= α (K δ - μ) / (K β δ)}, {B =
- K RootOf(K β δ σ Z^2 + (A α γ σ - K α δ σ + α μ σ) Z + A α γ θ) β δ - K α δ + μ α, P =
γ α
- K (RootOf(K β δ σ Z^2 + (A α γ σ - K α δ σ + α μ σ) Z + A α γ θ) β - α) / α, T = RootOf(K β δ σ Z^2
+ (A α γ σ - K α δ σ + α μ σ) Z + A α γ θ)}

> TK2 := TK[2];
      TK2 := {B=0, P=K, T=0}

> jac := Matrix(Jacobian([dP, dT, dB], [P, T, B]));
      jac := ⎡ α (1 - P/K) - α P/K - T β    -β P    0
              T δ    -B γ + P δ - μ    -T γ
              0    σ B (B/A - 1)    σ T (B/A - 1) + σ B T/A - θ ⎤

> jacTK2 := subs(TK2, jac);
      jacTK2 := ⎡ -α    -K β    0
                  0    K δ - μ    0
                  0    0    -θ ⎤

>
parameter := {α = 0.6, K = 100, β = 0.01, δ = 0.003, γ = 0.004, μ = 0.4, σ = 0.003, A = 10, θ = 0.05, P = P(t), T = T(t),
B = B(t)};
TitikStasioner := subs(parameter, TK2);

```

```
parameter := {A = 10, B = B(t), K = 100, P = P(t), T = T(t), α = 0.6, β = 0.01, δ = 0.003, γ = 0.004, μ = 0.4, σ = 0.003, θ = 0.05}
```

```
TitikStasioner := {B(t) = 0, P(t) = 100, T(t) = 0}
```

```
> MatrixTK2 := subs(parameter, jacTK2);
```

$$MatrixTK2 := \begin{bmatrix} -0.6 & -1.00 & 0 \\ 0 & -0.100 & 0 \\ 0 & 0 & -0.05 \end{bmatrix}$$

```
> CharaTK2 := simplify(factor(CharacteristicPolynomial(MatrixTK2, λ)));
```

$$CharaTK2 := (\lambda + 0.6) (\lambda + 0.1) (\lambda + 0.05)$$

```
> eigenTK2 := fsolve(CharaTK2, λ, complex);
```

$$eigenTK2 := -0.6000000000, -0.1000000000, -0.0500000000$$

```
>
```

```
sis := diff(P(t), t) = subs(parameter, dP), diff(T(t), t) = subs(parameter, dT), diff(B(t), t) = subs(parameter, dB);
```

$$\begin{aligned} sis := \frac{d}{dt} P(t) &= 0.6 P(t) \left(1 - \frac{1}{100} P(t)\right) - 0.01 P(t) T(t), \frac{d}{dt} T(t) = -0.004 B(t) T(t) + 0.003 P(t) T(t) \\ &\quad - 0.4 T(t), \frac{d}{dt} B(t) = 0.003 B(t) T(t) \left(\frac{1}{10} B(t) - 1\right) - 0.05 B(t) \end{aligned}$$

```
> inicial := P(0) = 80, T(0) = 10, B(0) = 6;
```

$$inicial := P(0) = 80, T(0) = 10, B(0) = 6$$

```
> sol := dsolve({sis, inicial}, numeric);
```

```
sol := proc(x_rkf45) ... end proc
```

```
> plotP := odeplot(sol, [t, P(t)], 0..150, color = green);
```

```
> plotT := odeplot(sol, [t, T(t)], 0..150, color = red);
```

```
> plotB := odeplot(sol, [t, 7 · B(t)], 0..150, color = blue);
```

```
> display(plotP, plotT, plotB);
```

Lampiran 2. Simulasi Numerik Titik Ekuilibrium Ketiga

```
> restart :
```

```
>
```

```
with(linalg) : with(LinearAlgebra) : with(plots) : with(Student[VectorCalculus]) : with(DEtools) : with(plots, implicitplot) : with(Student[CalculusI]) :
```

```
> dP := α · P · (1 - P/K) - β · P · T; dT := δ · P · T - γ · B · T - μ · T; dB := σ · B · T · (B/A - 1) - θ · B;
```

$$dP := \alpha P \left(1 - \frac{P}{K}\right) - \beta P T$$

$$dT := -B T \gamma + P T \delta - T \mu$$

$$dB := \sigma B T \left(\frac{B}{A} - 1\right) - \theta B$$

```
> TK := solve({dP, dT, dB}, {P, T, B});
```

$$\begin{aligned}
TK &:= \{B=0, P=0, T=0\}, \{B=0, P=K, T=0\}, \left\{B=-\frac{\mu}{\gamma}, P=0, T=-\frac{\theta A \gamma}{\sigma(A \gamma + \mu)}\right\}, \left\{B=0, P=\frac{\mu}{\delta}, T\right. \\
&= \left.\frac{\alpha(K \delta - \mu)}{K \beta \delta}\right\}, \left\{B=\right. \\
&= \frac{K \text{RootOf}(K \beta \delta \sigma _Z^2 + (A \alpha \gamma \sigma - K \alpha \delta \sigma + \alpha \mu \sigma) _Z + A \alpha \gamma \theta) \beta \delta - K \alpha \delta + \mu \alpha}{\gamma \alpha}, P= \\
&= \frac{K (\text{RootOf}(K \beta \delta \sigma _Z^2 + (A \alpha \gamma \sigma - K \alpha \delta \sigma + \alpha \mu \sigma) _Z + A \alpha \gamma \theta) \beta - \alpha)}{\alpha}, T = \text{RootOf}(K \beta \delta \sigma _Z^2 \\
&+ (A \alpha \gamma \sigma - K \alpha \delta \sigma + \alpha \mu \sigma) _Z + A \alpha \gamma \theta) \}
\end{aligned}$$

> TK4 := TK[4];

$$TK4 := \left\{B=0, P=\frac{\mu}{\delta}, T=\frac{\alpha(K \delta - \mu)}{K \beta \delta}\right\}$$

> jac := Matrix(Jacobian([dP, dT, dB], [P, T, B]));

$$jac := \begin{bmatrix} \alpha \left(1 - \frac{P}{K}\right) - \frac{\alpha P}{K} - T \beta & -\beta P & 0 \\ T \delta & -B \gamma + P \delta - \mu & -T \gamma \\ 0 & \sigma B \left(\frac{B}{A} - 1\right) & \sigma T \left(\frac{B}{A} - 1\right) + \frac{\sigma B T}{A} - \theta \end{bmatrix}$$

> jacTK4 := subs(TK4, jac);

$$jacTK4 := \begin{bmatrix} \alpha \left(1 - \frac{\mu}{\delta K}\right) - \frac{\mu \alpha}{\delta K} - \frac{\alpha(K \delta - \mu)}{K \delta} & -\frac{\beta \mu}{\delta} & 0 \\ \frac{\alpha(K \delta - \mu)}{K \beta} & 0 & -\frac{\alpha(K \delta - \mu) \gamma}{K \beta \delta} \\ 0 & 0 & -\frac{\sigma \alpha(K \delta - \mu)}{K \beta \delta} - \theta \end{bmatrix}$$

>

parameter := {α=0.5, K=100, β=0.01, δ=0.005, γ=0.001, μ=0.1, σ=0.002, A=5, θ=0.05, P=P(t), T=T(t), B=B(t)};

TitikStasioner := subs(parameter, TK4);

parameter := {A=5, B=B(t), K=100, P=P(t), T=T(t), α=0.5, β=0.01, δ=0.005, γ=0.001, μ=0.1, σ=0.002, θ=0.05}

$$TitikStasioner := \{B(t) = 0, P(t) = 20.00000000, T(t) = 40.00000000\}$$

> MatrixTK4 := subs(parameter, jacTK4);

$$MatrixTK4 := \begin{bmatrix} -0.1000000000 & -0.2000000000 & 0 \\ 0.2000000000 & 0 & -0.0400000000 \\ 0 & 0 & -0.1300000000 \end{bmatrix}$$

> CharaTK4 := simplify(factor(CharacteristicPolynomial(MatrixTK4, λ)));

$$CharaTK4 := (\lambda + 0.130000000000000004) (\lambda^2 + 0.100000000000000006 \lambda + 0.040000000000000000)$$

> eigenTK4 := fsolve(CharaTK4, λ, complex);

$$eigenTK4 := -0.1300000000, -0.05000000000 - 0.1936491673 I, -0.05000000000 + 0.1936491673 I$$

>

sis := diff(P(t), t) = subs(parameter, dP), diff(T(t), t) = subs(parameter, dT), diff(B(t), t) = subs(parameter, dB);

$$\begin{aligned} \text{sis} := \frac{d}{dt} P(t) &= 0.5 P(t) \left(1 - \frac{1}{100} P(t) \right) - 0.01 P(t) T(t), \quad \frac{d}{dt} T(t) = -0.001 B(t) T(t) + 0.005 P(t) T(t) \\ &- 0.1 T(t), \quad \frac{d}{dt} B(t) = 0.002 B(t) T(t) \left(\frac{1}{5} B(t) - 1 \right) - 0.05 B(t) \end{aligned}$$

```
> inicial := P(0) = 80, T(0) = 10, B(0) = 6;
                                inicial := P(0) = 80, T(0) = 10, B(0) = 6

> sol := dsolve( {sis, inicial}, numeric);
                                sol := proc(x_rkf45) ... end proc

> sol := dsolve( {sis, inicial}, numeric);
                                sol := proc(x_rkf45) ... end proc

> plotP := odeplot(sol, [t, P(t)], 0..150, color=green);
> plotT := odeplot(sol, [t, T(t)], 0..150, color=red);
> plotB := odeplot(sol, [t, 7·B(t)], 0..150, color=blue);
> display(plotP, plotT, plotB);
```

Lampiran 3. Syntax Model tanpa diterapkan Kontrol

```
function yt = PTB_TK(t,y)
%Nilai Parameter
alpha = 0.5; %Laju pertumbuhan padi
beta = 0.01; %Laju interaksi padi dan tikus
delta = 0.005; %Laju interaksi tikus dan padi
gamma = 0.001; %laju interaksi tikus dan burung hantu
mu = 0.1; %Laju kematian alami tikus
sigma = 0.002; %laju burung hantu dan tikus
theta = 0.05; % laju kematian alami burung hantu
K = 100; %carring capacity
A = 5; % allee effect

%Model Matematika
yt = zeros(3,1);
yt(1) = alpha*y(1)*(1-(y(1)/K)) - (beta*y(1)*y(2));
%P
yt(2) = (delta*y(1)*y(2)) - gamma*y(3)*y(2) -
mu*y(2); %T
yt(3) = sigma*y(3)*y(2)*((y(3)/A)-1) - theta*y(3); %B
```

Lampiran 4. Syntax Simulasi Numerik Model dengan diterapkan Kontrol

```
clear all;
close all;
clc;
%Nilai parameter
alpha = 0.5; %Laju pertumbuhan padi
beta = 0.01; %Laju interaksi padi dan tikus
delta = 0.005; %Laju interaksi tikus dan padi
```

```

gamma = 0.001; %laju interaksi tikus dan burung hantu
mu = 0.1; %Laju kematian alami tikus
sigma = 0.002; %laju burung hantu dan tikus
theta = 0.05;%Laju kematian alami burung hantu
K = 100; %carring capacity
A= 5;% allee effect

%Nilai awal
P0 = 80;
T0 = 10;
B0 = 6;

tfinal = 100; %waktu akhir pengamatan
C1 = 1;
C2 = 5;

y1 =
PTB_u(alpha,beta,delta,gamma,mu,sigma,theta,K,A,C1,C2
,P0,T0,B0,tfinal);

options = odeset ('RelTol',1e-4,'AbsTol',[1e-4 1e-4
1e-4]);
t0 = 0;
y0 = [P0, T0, B0];

disp(' ')
T1 = linspace(t0,tfinal,1001);
[t,y]=ode45('PTB_TK',T1,y0,options);

%Plot populasi tikus
figure(1)
plot(t,y(:,2),'r', y1(1,:),y1(3,:), '-
b','LineWidth',2)
xlabel('t(hari)')
ylabel('Populasi Tikus')
legend('tanpa diterapkan kontrol', 'diterapkan
kontrol u')

%plot profil kontrol
figure(2)
plot(y1(1,:),y1(5,:), '-b','LineWidth',2)
xlabel('t(hari)')
ylabel('profil kontrol')
legend('u')
axis([0 tfinal 0 1])

%Plot populasi padi

```

```

figure(3)
plot(t,y(:,1),'r', y1(1,:),y1(2,),'-
b','LineWidth',2)
xlabel('t(hari)')
ylabel('Populasi Padi')
legend('tanpa diterapkan kontrol', 'diterapkan
kontrol u')

%Plot populasi burung hantu
figure(4)
plot(t,y(:,3),'r', y1(1,:),y1(4,),'-
b','LineWidth',2)
xlabel('t(hari)')
ylabel('Populasi Burung Hantu')
legend('tanpa diterapkan kontrol', 'diterapkan
kontrol u')

%perhitungan indeks performansi
H = C1*y1(3,:) + 0.5*C2*y1(5,:).^2;
T = y1(1,:);
J1 = trapz(T,H);

```